



Bet You Can't Hit Me! The Science of Catapult Statistics

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Experimental Procedure

1. Set up the catapult.
 - a. Use the clamp to attach the catapult to the edge of a sturdy surface, like a table. Make sure you have plenty of open space (at least 12 feet [ft.]) in front of the catapult to launch the ball.
 - b. Lay your tape measure out on the floor in front of the catapult (in the same direction you will be launching the ball), with the end reading "zero" inches lined up with the center of the catapult's disk. Once you have placed the tape measure on the floor, you may want to tape it to the floor. You do not want it to move during your experiments; otherwise, measurements will not be consistent.

Video instructions for using your ping pong catapult.

<https://www.youtube.com/watch?v=pIEjwMhnAGo> (<https://www.youtube.com/watch?v=pIEjwMhnAGo>)

2. Select launch parameters for the catapult. Pick a launch angle, pull-back angle, type of ball, and number of rubber bands. Almost any settings you choose should work, but make sure they are sufficient to launch the ball a distance of at least a couple of feet. For example, a small pull-back angle of only 5 or 10 degrees, with just one rubber band, probably will not launch the ball very far, and this will make it difficult to take measurements (for example, if the ball doesn't even clear the base of the catapult).
3. Launch the ball. One person should be operating the catapult and ensuring that all the settings selected in Step 2 are correct. The second person should be sitting or kneeling along the side of the tape measure (how far away from the catapult will depend on the settings you've selected). This person should **pay very close attention to where the ball first hits the ground**. Immediately mark this location with your finger (rather than just trying to remember it) — you do **not** want to record where the ball comes to a final stop after rolling or bouncing. Be sure to write this distance down.
4. Now you are ready to start the data-collecting part of the project. Launch the ball at least 50 more times, with *exactly the same settings you used in step 2*.
 - a. The most difficult setting to repeat is the pull-back angle, because it is set by hand and can be adjusted continuously (as opposed to the launch angle, which is set by the pin shown in Figure 4 and can be adjusted in 15 degree increments only). So, whoever is operating the catapult must be careful to use exactly the same pull-back angle each time.
 - b. Remember to record **where the ball first hits the ground** and not how far it bounces or rolls. You can round to the nearest inch when measuring, as it will be difficult to accurately measure to within fractions of an inch. It may help to record your results with tick marks in a table (this will make it easier to make a histogram later). See Table 1 for an example.
 - c. If you have the time and patience, you can certainly do more than 50 trials. The more data you have, the better your histograms will be.

Distance (in inches)	Number of Trials
60	
61	
62	
63	
64	
65	
66	
67	
68	
69	
70	

Table 1. This table is one example of how you can record data. In the first column, mark distances, and in the second column, record the number of launches where the ball traveled that distance. Remember that the ball does not have to go *exactly* that distance: How you mark it will depend on how accurately you can measure. For example, a ball might actually travel 59.9 inches or 60.1 inches, but you would check both of those distances as 60 inches by rounding to the nearest whole number.

5. Calculate the mean and standard deviation of your data. You can do this by hand or with a spreadsheet program such as Microsoft Excel®.
 - a. To calculate the mean \bar{x} by hand, add up all the values and then divide by the total number of trials. For example (using five trials to keep the example short), say your measured distances were 52, 54, 57, 57, and 61 inches. The mean is the sum of all the distances divided by the number of trials: the numbers add up to 281, and there are five trials — so, dividing 281 by 5 gives the mean of 56.2:

Equation 1:

$$\bar{x} = \frac{52 + 54 + 57 + 57 + 61}{5} = \frac{281}{5} = 56.2$$

(Note: Hover over the equations in this Procedure with your cursor to view enlarged formulas.)

In general, the equation to calculate the mean is

Equation 2:

$$\bar{x} = \frac{\sum_{i=1}^N x_i}{N}$$

- **N** is the total number of trials
- **i** is the *index* used to keep track of each trial, starting at 1 and going up to *N*
- **x_i** is the value for a trial at index *i*
- **\bar{x}** is the mean

b. (Skip this step if you are not using Excel®). To calculate the mean in Excel®, enter all of the values (52, 54, 57, 57, 61) in a single column or row, and then use the AVERAGE() command. For example, if you typed your values into column A in rows 1 through 5, you would type **=AVERAGE(A1:A5)** into any other cell to calculate the mean.

Technical Note:

We stated in the Introduction that "mean" and "average" are sometimes used interchangeably, even though this is not technically correct. Here is a perfect example — Excel® uses the command AVERAGE() to calculate the mean. It has separate commands for MEDIAN() and MODE().

c. The formula to calculate standard deviation by hand is a bit more complicated. It might look scary at first, but do not worry. It is actually not that hard to calculate. The formula for standard deviation is:

Equation 3:

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2}$$

- **N** is the total number of trials
- **i** is the *index* used to keep track of each trial, starting at 1 and going up to *N*
- **x_i** is the value for a trial at index *i*
- **\bar{x}** is the mean
- **s** is the standard deviation

We will break this down into steps using our example data to make it easier to understand. The first thing to do is take the difference between each of the individual values and the mean you calculated — that is, the $(x_i - \bar{x})$ part of the equation. So, for our data, (52, 54, 57, 57, 61), and our mean, 56.2, that comes out to:

- $52 - 56.2 = -4.2$
- $54 - 56.2 = -2.2$
- $57 - 56.2 = 0.8$
- $57 - 56.2 = 0.8$
- $61 - 56.2 = 4.8$

Next you have to *square* each one of those numbers — notice how the $(x_i - \bar{x})$ part of the equation is squared. So, we have:

- $-4.2^2 = 17.64$
- $-2.2^2 = 4.84$
- $0.8^2 = 0.64$
- $0.8^2 = 0.64$
- $4.8^2 = 23.04$

Now we have to add all of those numbers up — that is what the \sum symbol means. Now we get:

- $17.64 + 4.84 + 0.64 + 0.64 + 23.04 = 46.8$

Almost done! Notice the $1/(N-1)$ inside the square root. So now we have to divide this number by $N-1$ (in this case $N = 5$):

- $46.8/(5-1) = 11.7$

And finally, we take the square root of that number to solve for *s*:

$$s = \sqrt{11.7} \approx 3.42$$

d. (Skip this step if you are not using Excel®). You can also calculate the standard deviation in Excel® using the STDEV() function. This will be easier than doing it by hand for a very large amount of data. For example, if you enter your five values into cells A1 through A5, you would type **=STDEV(A1:A5)** in another cell to calculate the standard deviation.

Technical Note:

There is a slight difference in the calculation for standard deviation if you are using an entire population instead of just a sample. Here we are

assuming that your data is just a sample of all possible shots that can be taken with the catapult, because you can always do more trials. For an entire population, the equation is

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \mu)^2}$$

The only practical difference, aside from changing the letters used for the variables, is that the (N-1) in the denominator changed to just an N. We will not go into the details for this here, but if you are curious, search for "Bessel's Correction" online.

6. By now you should have run through at least 50 trials for one set of catapult settings (launch angle, pull-back angle, ball type, and number of rubber bands), recorded the distance for each launch, and calculated the mean and standard deviation. For your next step, *find new catapult settings that launch the ball about the same distance*. For example:
 - Say you were using two rubber bands to launch the ball about 70 inches. Can you do this with just one rubber band by changing the other parameters?
 - If you were hitting a certain distance with a 30-degree launch angle, can you also hit that distance with a 75-degree launch angle? How?
 - Finding the new settings will take a bit of trial and error.
7. Repeat steps 3 through 5 for these new launch settings.
8. Compare the mean and standard deviation for each of your experiments. Does one set of data have a smaller standard deviation? If so, this indicates that those settings were more reliable. Why do you think that happened?
9. Construct histograms for both of your data sets. Remember that the standard deviation will correspond to how "spread out" the histogram looks. You may also need to adjust the **bin size** of your histogram. For example, say that for more than 50 shots, your shortest distance was 58 inches and your longest was 72 inches. Your histogram might not look right if you have a bar for every one-inch increment — it may be better to have larger "bins", such as 58 to 61 inches, 61 to 64 inches, 64 to 67 inches, and so forth. If you need more help understanding this concept, refer to Science Buddies' [Catapult FAQ](http://www.sciencebuddies.org/science-fair-projects/project-ideas/Phys_p085/physics/use-a-catapult-to-storm-castle-walls) (http://www.sciencebuddies.org/science-fair-projects/project-ideas/Phys_p085/physics/use-a-catapult-to-storm-castle-walls).
10. Does your data look normally distributed? Are your histograms symmetric, or skewed to one side? Can you explain why they look the way they do?

Frequently Asked Questions (FAQ)

FAQ for this Project Idea available online at https://www.sciencebuddies.org/science-fair-projects/project-ideas/Math_p046/pure-mathematics/science-of-catapult-statistics#help (http://www.sciencebuddies.org/science-fair-projects/project-ideas/Math_p046/pure-mathematics/science-of-catapult-statistics#help).