Launch Time: The Physics of Catapult Projectile Motion

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Experimental Procedure

Physical Constants

Before starting your experiments, you will need to measure or calculate several physical constants necessary for your theoretical results. You will need:

- The mass of the ping pong ball, m
- The radius from the center of rotation to the center of the ping pong ball, r
- The length of the launch arm, L
- The moment of inertia of the launch arm, I
- The unstretched length of a rubber band, x_0
- The spring coefficient of a rubber band, k. (We use the word "coefficient" instead of "constant" since the rubber bands are nonlinear springs).

1. In case you do not have a scale, we will provide the mass of the ball for you (this was for the orange ping pong ball included with the catapult kit in September 2012; if this changes in the future, you will need to measure the mass of the new ball). We determined that m = 2.7 grams (g).
2. You should be able to measure the lengths L and r easily with a ruler; see Figure 5 in the Introduction.
3. The arm's moment of inertia I is a bit more complicated, as it cannot be measured directly with an instrument like a ruler or a scale. It must be calculated; we arrived at a value of I = 1.1 x 10^{-3} kg*m².
   Advanced students: Can you do this calculation yourself, and include it as part of your project? Assume the rod is made out of aluminum. Do different formulas for moment of inertia give you different results (for example, assuming the arm is a "thin rod", or a combination of rectangular prisms)?
4. For the rubber band, we recommend measuring your own values in case the type of rubber bands shipped with the catapult kit change over time. To measure x_0, you will have to measure the length of the rubber band while it is *unstretched* but *flat*. Figure 6 shows how to do this.
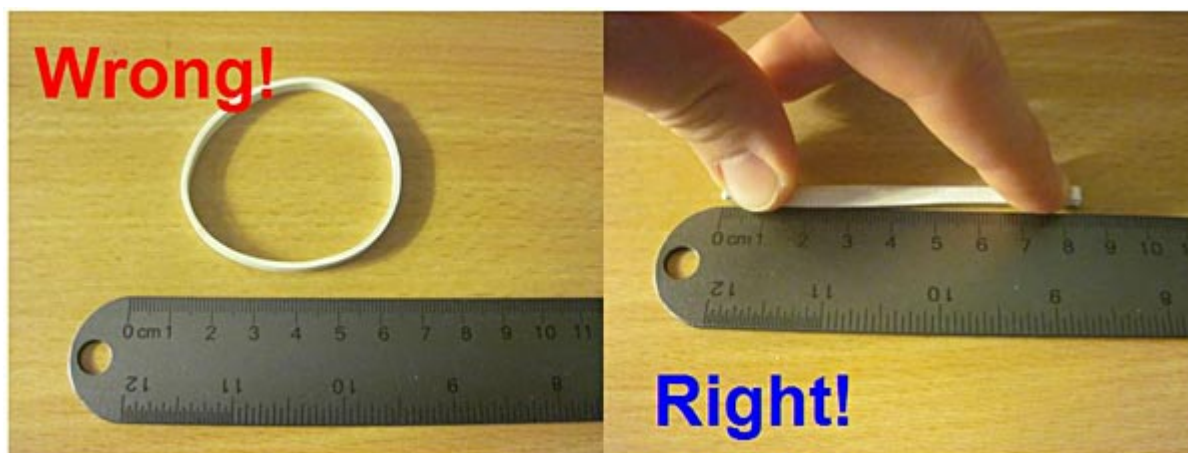

Figure 6. The incorrect (left) and correct (right) ways to measure the unstretched length of the rubber band, x_0. Note that in the image on the right, the rubber band is NOT being stretched — it is just pressed flat.

5. Measuring the spring coefficient k is slightly more complicated, because the rubber bands do not behave as linear springs. We found the value in Equation 15 (from the Introduction) to be k = 33.55 N/m^{½}. (Note that for a linear spring, k always has units of N/m — in this case the units are different because the spring is nonlinear).

Advanced students: Can you calculate this value yourself and include it as part of your project?

Theoretical Predictions

1. Select a launch angle, pull-back angle, and number of rubber bands that you would like to test.
   a. Using Equations 8 and 10 from the Introduction, fill out a table or spreadsheet of the expected position of the ball at different times (remember that first you will have to use Equation 16 to calculate the launch velocity). We recommend starting at time $t=0$ and going up to $t=1$ second (the ball is usually in the air for less than a second, so this should give you more than enough theoretical data), in intervals of 1/30th of a second (this will correspond to the frame rate of most video cameras).
   b. This will be easiest with a spreadsheet program such as Microsoft Excel®.
   c. You can add your experimental results to this spreadsheet later, to make it easier to compare the two, as in Table 1.

| | Theoretical | | Experimental | |
Time (seconds)	x (meters)	y (meters)	x (meters)	y (meters)
0.000				
0.033				
0.067				
0.100				
0.133				
0.167				
0.200				

Table 1. A data table such as this enables you to gather theoretical predictions as well as experimental measurements and compare the two.

Filming the Ball

1. Watch this video to learn how to set up your catapult:

> Video instructions for using your ping pong catapult.
> https://www.youtube.com/watch?v=pIEjwMhnAGo (https://www.youtube.com/watch?v=pIEjwMhnAGo)

2. Next you will need to find an open, well-lit area to film a launch, using the same catapult settings you just used for your theoretical predictions. Make sure your camera is far enough away that you can see the catapult itself, and that the ball does not go off-screen before it hits the ground. If you can adjust settings on your camera, like the exposure time or frame rate, you may need to take several videos to find the best settings. Your goal is to be able to clearly see the location of the ball in each frame, without blurring. *Important*: You will need an object with a known length (such as a ruler) in the video for scale. This will enable you to convert the distance you measure on your computer screen to real-world distance.
3. Also remember to measure and record the length of the rubber band when the launch arm is pulled back to your desired pull-back angle, and at launch (when the arm is all the way up against the rubber stopper). Remember that you need to measure the *total* length of the rubber band, which is folded in half.

4. To make sure that your launch is not a fluke (which could really throw you off when you compare your experimental results to your theoretical data), we recommend filming at least three launches with the same settings.

Analyzing your Videos

1. Upload your videos to a computer, and open them in a program that can play them frame-by-frame (we recommend Apple QuickTime®, as other players often do not let you step through videos one frame at a time). Using a ruler, measure the x and y positions of the ball in each frame, starting from the moment the ball leaves the launch cup (be careful not to scratch your computer screen with the ruler). Remember that in the Introduction, we defined the ground as y=0, with the ball leaving the launch cup at height *h* above the ground, and the ball starting off at x=0.
   - Optional: instead of measuring the ball's position on your screen with a ruler, you can open your video file in a program like Tracker Video Analysis and Modeling (https://physlets.org/tracker/), which will allow you to click on the ball in each frame of your video to record its location. You will need to read the instructions for the Tracker program and learn how to use it.
2. *Important*: Remember that the distances you measure on your computer screen are not the same as the real-life position of the ball! You should have had a scale in your videos — we will use a 30-centimeter (cm) ruler as an example. Say a 30 cm ruler in your video appears to be 3 cm long on your computer screen. This means that your video has a scale of 30:3, or 10:1. So, to convert your on-screen measurements, to real-life values, you multiply all of them by 10. For example:
   - You measure the (x,y) position of the ball on your computer screen to be x = 4 cm, y = 2 cm.
   - Multiply each of these values by 10 to get the actual position of the ball: x = 40 cm, y = 20 cm.
   - Remember that your video might have a different scale factor! This is just an example.
   - The scale factor will change if you change the size of the window on your computer screen. If you open the program, calculate your scale factor, and then make the video full-screen, your scale factor will no longer be valid.

Compare your Results

Enter your experimental results from your video analysis in the same table or spreadsheet as your theoretical results. Now you can plot the (x,y) position of the ball for both theoretical and experimental results on the same graph. Are they the same?

If your experimental and theoretical results are different, can you explain why? Can you do multiple trials with different launch settings to see if some match up better than others?

Frequently Asked Questions (FAQ)

FAQ for this Project Idea available online at https://www.sciencebuddies.org/science-fair-projects/project-ideas/Phys_p089/physics/physics-of-catapult-projectile-motion#help.