

1. Consider a spherical planet of mass M and surface radius R_s . The planet has no atmosphere, and you may assume that any object in a vicinity of the planet is subject to the gravitational force only.
- (a) One common method of studying properties of a planet's surface is by deliberately crashing a probe into it and inspecting the characteristics of the crater that appears after the impact. You set a probe of mass m into a free fall with zero initial velocity a distance R away from the centre of the planet ($R > R_s$). Find the kinetic energy of the probe just before the impact. You may neglect any changes of the planet's momentum during this process. (4)
- (b) The probe comes to rest in a short time interval of the impact Δt . What is the average force exerted on the surface of the planet during this impact? (2)
- (c) A satellite of mass m is set into a constant speed circular orbit of radius R around the planet. What is the angular velocity ω_0 and speed V_0 of the satellite? What is the angular momentum L_0 of the satellite relative to the centre of the planet? Show on a diagram vectors \vec{V}_0 and \vec{L}_0 . (4)
- (d) At time $t = 0$ the satellite's engine is switched on to initiate a descent towards the surface of the planet in a spiral trajectory, such that the angular velocity of the satellite is kept constant $\omega = \omega_0$, and the orbital radius is decreasing at a constant rate $\dot{r} = u < 0$. What must be the *tangential component* F_θ of the force exerted by the satellite's engine to support this manoeuvre? (4)
- (e) Consider a short initial stage of the spiral trajectory of the satellite, when the variation of the radial distance is relatively small: $r(t) = R + \Delta r(t)$, $\Delta r(t) \ll R$. Show that the *radial component* F_r of the force exerted by the satellite's engine during this stage can be approximated as:

$$F_r \approx -\frac{3GMm}{R^3} ut.$$